Chapter 4 lecture questions

Q1: If you have an instrument taking one measurement every second for 24 hours. What are (a) the fundamental frequency ω_1, (b) the Nyquist frequency ω_N and (c) $\Delta\omega$, the frequency between two adjacent frequency bands?

Answer:
(a) $\omega_1 = 7.27 \times 10^{-5} \text{s}^{-1}$, (b) $\omega_N = 3.14 \text{ s}^{-1}$ and (c) $\Delta\omega = 7.27 \times 10^{-5} \text{ s}^{-1}$.

Solution:
(a) With $T = 24$ hr $= 86400$ s, $\omega_1 = \frac{2\pi}{T} = \frac{2\pi}{86400} = 7.27 \times 10^{-5} \text{ s}^{-1}$.

(b) With $\Delta t = 1$ s, $\omega_N = \frac{\pi}{\Delta t} = \frac{3.1415}{1} = 3.14 \text{ s}^{-1}$.

(c) $\Delta\omega = \frac{2\pi}{T} = \omega_1$, hence the same answer as in (a).

Q2: Suppose the data are dominated by grid-scale noise, i.e. the data in 1-dimension have adjacent grid points simply flipping signs like $\ldots, -1, +1, -1, +1, -1, +1, \ldots$, what happens (a) when you apply the 3-point moving-average filter to the data once? twice? (b) when you apply the triangular filter to the data once? twice?

Solution:
(a) Apply $\tilde{x}_n = \frac{1}{3}x_{n-1} + \frac{1}{3}x_n + \frac{1}{3}x_{n+1}$ to
$\ldots, -1, +1, -1, +1, -1, +1, \ldots$,
we get $\ldots, 1/3, -1/3, +1/3, -1/3, +1/3, -1/3, \ldots$.
Apply the filter a second time gives:
$\ldots, 1/9, -1/9, +1/9, -1/9, +1/9, -1/9, \ldots$.

(b) Apply $\tilde{x}_n = \frac{1}{4}x_{n-1} + \frac{1}{2}x_n + \frac{1}{4}x_{n+1}$ to
$\ldots, -1, +1, -1, +1, -1, +1, \ldots$,
we get $\ldots 0, 0, 0, 0, 0, 0, \ldots$.
Apply the filter a second time gives:
$\ldots 0, 0, 0, 0, 0, 0, \ldots$.
Hence the triangular filter is much more effective than the 3-point moving average filter in eliminating grid-scale noise.

Q3: (a) What is the dimension of the augmented data matrix $\mathbf{Y}$ with lag L, where the original data matrix has m variables and n time observations?
(b) If your spatial domain has 1°lat.×1°lon. gridded data covering 50° of latitude and 130°

of longitude, and you have monthly observations for 60 years, what is the dimension of your matrix $\mathbf{Y}$ with lag $L = 72$ months?

Answer:
(a) Dimension of $\mathbf{Y} = Lm \times (n - L + 1)$
(b) Dimension of $\mathbf{Y} = 468,000 \times 649$

Solution:

$$\mathbf{Y} = \begin{bmatrix} y_{11} & y_{12} & \cdots & y_{1,n-L+1} \\ \vdots & \vdots & \vdots & \vdots \\ y_{m1} & y_{m2} & \cdots & y_{m,n-L+1} \\ \vdots & \vdots & \vdots & \vdots \\ y_{1L} & y_{1,L+1} & \cdots & y_{1n} \\ \vdots & \vdots & \vdots & \vdots \\ y_{mL} & y_{m,L+1} & \cdots & y_{mn} \end{bmatrix}. \tag{1}$$

(a) There are $n-L+1$ columns, and Lm rows, since the original data matrix was repeated L times. Hence the dimension of $\mathbf{Y} = Lm \times (n - L + 1)$.
(b) Total number of spatial variables $= m = 50 \times 130 = 6500$. With 12 observations per year and 60 years, the total number of time points $= n = 60 \times 12 = 720$. Dimension of $\mathbf{Y} = (Lm) \times (n - L + 1) = (72 \times 6500) \times (720 - 72 + 1) = 468,000 \times 649$.